BIULETYN WAT
VOL. XLIII, No 7-8, 1994

Analysis of a temperature distribution of a laser welding of metals

WOJCIECH PRZETAKIEWICZ***
SLAWOMIR KACZMAREK**
JOZEF RAFA*
ANDRZEJ PAWLATA**

* Institute of Mathematics and Operations Research,
** Institute of Quantum Electronics,
*** Deputy Commandant of the Military University of Technology 01-489
Warsaw, S. Kaliskiego 2, POLAND

Abstract. In this paper the theoretical model for laser welding was shown. It was stated the boundary problems for heat transfer equation in the range of material layer with finite thickness for moving laser source. It was currying out the nondimensional analysis, where the problem solution was stated by three dimension]ess *parameters. The.* constructed solution will be used for optimization and expenmental v-erification under *the* .laser treatment processes.; . .

AMS subject classification (1991): 35L40, 35E05, 35E15
Key words: 3D temperature distribution problem, moving heat source layer with finite thickness, boundary conditions for heat transfer with external exchange, Fourier transform method, analytical solution.

1. Introduction

Lasers with high values of power are often used in industry for laser welding, laser cutting, laser hardening and laser alloying by means of method [2-5, 7, 11-13,15, 16, 18]. In [1, 6] it is analyzed the above problem on the basis of temperature distribution model for heat equation with a moving source. Usually the mathematical models are limited to linear sources, homogeneous sources or point sources [9]. These wasn't take into account finite sheet thickness and take the simple heat exchange conditions at the limiting surfaces of the sheet. In present paper we describe spatial problem of heat distribution within the layer with finite thickness, for source model in the form of Gaussian beam absorbed down the material. Boundary condition describes all heat exchange (mixed problem). It was obtained the solution of above problem using the integral Fourier transform method. The solution of the problem is represented by single integrals, which one can easy to calculate numerically.

2. Formulation of the problem

In the coordinate system connected with a source, the temperature distribution in a moving sheet is describing by the following equation [10]:

$$\rho c v \frac{\partial T}{\partial x} - k \Delta T = q_v, \tag{1}$$

$$T : (x, y, z) \to \mathbb{R}_+, \quad (x, y) \in \mathbb{R} \times \mathbb{R}, \quad z \in (0, L)$$

and boundary conditions:

$$\begin{aligned} k\frac{\partial T}{\partial z} &= h_z(T - T_0) \qquad \text{for } z = 0, \\ \frac{\partial T}{\partial z} &= 0 \qquad \text{for } z = L, \end{aligned} \tag{2}$$

$$\lim_{R\to\infty} (T - T_0) = 0, \quad \lim_{R\to\infty} |\operatorname{grad} T| = 0,$$

$$R = \sqrt{x^2 + y^2},$$

where:

T — temperature within a layer,
T_o — ambient temperature,
v — velocity of a sheet moving in the x direction,
p — mass density of a material,
c — heat capacity of a material,
k — heat conductivity of a material,
h_z — coefficient of external heat exchange; $h_z = h_e + h_r$,
q_v — spatial distribution of voluminal density of source power.

$$q_v = \frac{2P\varepsilon}{\pi W_0^2 L} f(x, y) \mathrm{e}^{-\gamma z}, \tag{3}$$

where:

$f(x, y)$ —distribution of the laser radiation,
P — beam laser power,
e — absorption ability of a material,
L — sheet thickness,

γ — coefficient of radiation absorption of a material,
2Wo — diameter of laser beam,
h_c = I3 $Re^{0.5}$ $Pr^{0.33}$ k_{gaz}/B [10], .;
B — distance from a nozzle to sheat,
Re — Rayleigh number,
Pr — Prandtl number,
k_{gaz} — heat conduction of gaz (argon),
$hr=\varepsilon\delta T_o^3$
δ — Stefan-Boltzmann constant.

Dimension analysis of the set of physical parameters permiss us to write the problem in the following dimensionless form:

$$\frac{\partial\theta}{\partial x_1} = \mathrm{V_e}\left(\frac{\partial^2\theta}{\partial x_1^2} + \frac{\partial^2\theta}{\partial x_2^2} + \frac{\partial^2\theta}{\partial x_3^2}\right) + m\mathrm{e}^{-\gamma_0 x_3} f(x_1, x_2), \tag{4}$$
$$\frac{\partial\theta}{\partial x_3} = Bi\theta, \quad x_3 = 0, \quad \lim_{|x|\to\infty} \theta = \lim_{|x|\to\infty} |\nabla\theta| = 0,$$
$$\frac{\partial\theta}{\partial x_3} = 0, \quad x_3 = L, \quad |x| = \sqrt{x_1^2 + x_2^2},$$

where dimensionless parameters are:

$\theta = (T-T_o)/T_m$ — dimensionless temperature,
T_m — melting temperature,
V_c=k/(rcvL) – Veron's number
Bi = $h_zL/k > 0$ — Biot number,
m = $P_\varepsilon/(\rho c T_m \pi W_o^2 v) > 0$ — dimensionless heat power,
$\gamma_o = \gamma L > 0$ — dimensionless absorption coefficient,
$\lambda = \sqrt{2}L/W_o > 0$ — dimensionless parameter of Gauss distribution,
x_1 = x/L, x_2 = y/L, x_3 = z/L— dimensionless coordinates.

3. Solution of the problem

We are looking the solution of the equation (4) in the following form:

$$\theta = \mathrm{e}^{\frac{x_1}{2\mathrm{V_e}}} \mathrm{e}^{-\gamma_0 x_3} \theta_1(x_0, x_2) + \mathrm{e}^{\frac{x_1}{2\mathrm{V_e}}} \theta_2(x_0, x_2, x_3), \tag{5}$$

where:

$x_0 = x_1 + \frac{1}{4\lambda^2 \mathrm{V_e}}$, $\theta_1(x_0, x_2)$ and $\theta_2(x_0, x_2, x_3)$

are the functions which satisfies the following equations:

$$\left(\gamma_0^2 - \frac{1}{4\mathrm{V_e^2}}\right)\theta_1 + \frac{\partial^2\theta_1}{\partial x_1^2} + \frac{\partial^2\theta_1}{\partial x_2^2} + \frac{m}{\mathrm{V_e}} \mathrm{e}^{\frac{1}{\gamma\lambda^2 \mathrm{V_e^2}}} \mathrm{e}^{\frac{x_0}{2\mathrm{V_e}}} f(x_0, x_2) = 0,$$
$$\left(-\frac{1}{4\mathrm{V_e^2}}\right)\theta_2 + \frac{\partial^2\theta_2}{\partial x_0^2} + \frac{\partial^2\theta_2}{\partial x_2^2} + \frac{\partial^2\theta_2}{\partial x_3^2} = 0 \tag{6}$$

and boundary conditions;

$$\begin{aligned}&\frac{\partial\theta_2}{\partial x_3}-(\gamma_0+\mathrm{Bi})\theta_2=(\gamma_0+\mathrm{Bi})\theta_1,\quad x_3=0,\\&\frac{\partial\theta_2}{\partial x_3}-\gamma_0\theta_2=\gamma_0\theta_1,\quad x_3=1.\end{aligned}\tag{7}$$

We shall to construct the solution of the problem by using the integral Fourier transforms with respect to x_0 and x_2 [8]:

$$\mathfrak{F}(x_0,x_2)\to(\alpha,\beta),\qquad [\theta_{1,2}]\overset{\mathrm{def}}{=}\int_{\mathbb{R}^2}\theta_{1,2}(x_0,x_2)\mathrm{e}^{-\mathrm{i}(\alpha x_0+\beta x_2)}\mathrm{d}x_0\mathrm{d}x_2\tag{8}$$

and

$$\mathfrak{F}(x_0,x_2)\to(\alpha,\beta),\quad \left[m_0\mathrm{e}^{\frac{x_0}{2\mathrm{V_e}}}f(x_0,x_2)\right]\overset{\mathrm{def}}{=}\int_{\mathbb{R}^2}m_0\mathrm{e}^{\frac{x_0}{2\mathrm{V_e}}}f(x_0,x_2)\mathrm{e}^{-\mathrm{i}(\alpha x_0+\beta x_2)}\mathrm{d}x_0\mathrm{d}x_2 \overset{\mathrm{def}}{=}\hat{g}(\alpha,\beta).\tag{9}$$

After the above transformations we obtain:

$$\hat{\theta}_1=\frac{\hat{g}}{\alpha^2+\beta^2+\frac{1}{4\mathrm{V_e^2}}-\gamma_0^2},\tag{10}$$

$$\hat{\theta}_2=A\mathrm{e}^{-x_3\sqrt{\alpha^2+\beta^2+\frac{1}{4\mathrm{V_e^2}}}}+B\mathrm{e}^{-|2-x_3|\sqrt{\alpha^2+\beta^2+\frac{1}{4\mathrm{V_e^2}}}},\tag{11}$$

$$\sqrt{\alpha^2+\beta^2+\frac{1}{4\mathrm{V_e^2}}}=\kappa,\qquad \Re\mathfrak{e}\kappa>0,$$

where A, B constans one can obtain from boundary conditions:

$$\begin{aligned}A&=\hat{\theta}_1\frac{(\mathrm{Bi}+\gamma_0)-\frac{\gamma_0\mathrm{e}^{-\gamma_0}}{\kappa}(\kappa-\mathrm{Bi})\mathrm{e}^{-\kappa}}{\Delta},\\B&=\hat{\theta}_1\frac{(\gamma_0+\mathrm{Bi})-\frac{\gamma_0(\kappa+\mathrm{Bi})}{\kappa}\mathrm{e}^{-(\kappa+\gamma_0)}}{\Delta},\\\Delta&=(\kappa-\mathrm{Bi})\mathrm{e}^{-2\kappa}-(\kappa+\mathrm{Bi}).\end{aligned}\tag{12}$$

After some calculations, we get:

$$\begin{aligned}\theta_1(x_0,x_2)&=\frac{1}{4\pi^2}\int_{\mathbb{R}^2}\frac{\hat{g}\mathrm{e}^{\mathrm{i}(\alpha x_0+\beta x_2)}}{\alpha_2^2+\beta_2^2+\frac{1}{4\mathrm{V_e^2}}-\gamma_0^2}\mathrm{d}\alpha\mathrm{d}\beta,\\\theta_2(x_0,x_2,x_3)&=\frac{1}{4\pi^2}\int_{\mathbb{R}^2}\hat{\theta}_2(\alpha,\beta,x_3)\mathrm{e}^{\mathrm{i}(\alpha x_0+\beta x_2)}\mathrm{d}\alpha\mathrm{d}\beta,\end{aligned}\tag{13}$$

On the case $\kappa^2_0 = 1/(4V^2_0) - \gamma_o^2 > 0$; the solution of the problem (after transformation) has the form:

$$\hat{\theta}_1 = \frac{m e^{\frac{1}{\gamma\lambda^2 V_e}}}{V_e}\frac{\hat{g}(\alpha,\beta)}{\alpha^2+\beta^2+\frac{1}{4V_e^2}-\gamma_0^2} = m_0\frac{\hat{g}}{\alpha^2+\beta^2+\kappa_0^2},$$
$$\kappa_0^2 = \left(\frac{1}{2V_e}\right)^2 - \gamma_0^2 > 0, \tag{14}$$
$$m_0 = \frac{m}{V_e}e^{-\frac{1}{\gamma_0\lambda^2 V_e}},$$
$$\hat{g}(\alpha,\beta) = \mathfrak{F}(x_0,x_2)\left[e^{-\frac{x_0}{2V_e}} f_1(x_0,x_2)\right],$$

$$f_1(x_0;x_2) = f\left(x_0 - \frac{1}{4\lambda^2 V_e}, x_2\right), \tag{15}$$

$$\hat{\theta}_2 = Ae^{-x_3\kappa} + Be^{-(2-x_3)\kappa}, \tag{16}$$

$$A = \left(-\frac{\hat{\theta}_1}{\kappa}\right)\frac{\kappa(\gamma_0+\mathrm{Bi}) - \gamma_0 e^{-(\kappa+\gamma_0)}(\kappa-\mathrm{Bi})}{(\kappa+\mathrm{Bi})-(\kappa-\mathrm{Bi})e^{-2\kappa}},$$
$$B = \left(-\frac{\hat{\theta}_1}{\kappa}\right)\frac{-(\kappa+\mathrm{Bi})\gamma_0 - e^{-(\gamma_0+\kappa)}(\gamma_0+\mathrm{Bi})}{(\kappa+\mathrm{Bi})-(\kappa-\mathrm{Bi})e^{-2\kappa}}.$$

So, we have:

$$\hat{\theta}_2 = \frac{\hat{\theta}_1}{\kappa\Delta}\left[\kappa(\gamma_0+\mathrm{Bi}) - \gamma_0(\kappa-\mathrm{Bi})e^{-(\gamma_0+\kappa)}\right]e^{-\kappa x_3} \tag{18}$$
$$+ \left[(\kappa+\mathrm{Bi})\gamma_0 e^{-\gamma_0} - \kappa(\gamma_0+\mathrm{Bi})e^{-\kappa}\right]e^{-(1-x_3)\kappa},$$
$$\Delta = (\kappa+\mathrm{Bi}) - (\kappa-\mathrm{Bi})e^{-2\kappa},$$
$$\theta_n = \frac{1}{4\pi^2}\int_{\mathbb{R}^2}\hat{\theta}_n e^{i(\alpha x_0+\beta x_2)}d\alpha d\beta, \quad n=1,2.$$

The function $\Theta0(x_1,x_2)$ can be represent in the form of the following convolution:

$$\theta_1 = \theta_{11} \underset{2}{*} g = \int_{\mathbb{R}^2}\theta_{11}(\xi_0,\xi_2)e^{\frac{-(x_0-\xi_0)}{V_e}} f(x_0-\xi_0-w, x_2-\xi_2)d\xi_0 d\xi_2, \tag{19}$$

where:

$$\theta_{11}(x_0,x_2) = \frac{m_0}{4\pi^2}\int_{\mathbb{R}^2}\frac{e^{i(\alpha x_0+\beta x_2)}}{\alpha^2+\beta^2+\kappa_0^2}d\alpha d\beta \tag{20}$$

and describes the general solution for equation (6). Using simple calculation we find that [17]:

$$\theta_{11}(x_0, x_2) = \frac{m_0}{2\pi}\mathrm{K}_0(\kappa_0\rho), \qquad \rho = \sqrt{x^2 + x^2}, \tag{21}$$

$K_o(.)$ — Me Donald's function.

So, we have:

$$\theta_1(x_0, x_2) = \frac{m_0}{2\pi}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\mathrm{K}_0\Big(\kappa_0\sqrt{(x_0-\xi_0)^2+(x_2-\xi_2)^2}\Big)\mathrm{e}^{\frac{\xi_0}{\mathrm{V_e}}}f(\xi_0 - w, \xi_2)\mathrm{d}\xi_0\mathrm{d}\xi_2. \tag{22}$$

Acting in the same way we can write the function Θ_2, as follows:

$$\theta_2(x_0, x_2, x_3) = \int\limits_{\mathbb{R}^2}\theta_1(\cdot)\theta_{21}\mathrm{d}\xi_0\mathrm{d}\xi_2 \tag{23}$$

where:

$$\theta_{21} = \frac{1}{4\pi^2}\int\limits_{\mathbb{R}^2}\frac{1}{\kappa\Delta}\Big[a\mathrm{e}^{-\kappa x_3} + b\mathrm{e}^{-\kappa(1-x_3)}\Big]\mathrm{e}^{\mathrm{i}(\alpha x_0+\beta x_2)}\mathrm{d}\alpha\mathrm{d}\beta \tag{24}$$

$$a = \kappa(\gamma_0 + \mathrm{Bi}) - \gamma_0(\kappa - \mathrm{Bi})\mathrm{e}^{-(\gamma_0+\kappa)}$$

$$b = (\kappa + \mathrm{Bi})\gamma_0\mathrm{e}^{-\gamma_0} - \kappa(\gamma_0 + \mathrm{Bi})\mathrm{e}^{-\kappa}$$

Assuming that the Gauss distribution of the laser radiation is given in the form:

$$f(x_1, x_2) = \mathrm{e}^{-\lambda^2(x_1^2+x_2^2)}, \tag{25}$$

we can write 6\ as follows:

$$\hat{\theta}_1 = \frac{m\mathrm{e}^{w^2}}{2\pi\mathrm{V_e}\lambda^2}\frac{\mathrm{e}^{\frac{-(\alpha^2+\beta^2)}{\lambda^2}}}{\alpha^2+\beta^2+\kappa_0^2}, \tag{26}$$

$$\theta_1 = m_1\int\limits_{\mathbb{R}^2}\frac{\mathrm{e}^{\frac{-(\alpha^2+\beta^2)}{\lambda^2}}\mathrm{e}^{\mathrm{i}(\alpha x_0+\beta x_2)}}{\alpha^2+\beta^2+\kappa_0^2}\mathrm{d}\alpha\mathrm{d}\beta, \tag{27}$$

where: $m_1 = 2m\exp(w^2)/\pi w^{-1}$, $w = 1/(4\lambda^2 V_e)$

Applying De'Hoop transformation [8]:

$$\alpha x_0 + \beta x_2 = \rho\omega,$$

$$-\alpha x_2 + \beta x_0 = \rho q,$$

we obtain:

$$\theta_1 = m_1 \int\limits_{\mathbb{R}^2} \frac{e^{\frac{-(\omega^2+q^2)}{\lambda^2}} e^{i\omega\rho}}{\omega^2 + q^2 + \kappa_0^2} d\omega dq. \tag{28}$$

Putting: $\omega = r\cos\varphi$, $q = r\sin\varphi$, $r = \sqrt{\omega^2 + q^2}$ we have:

$$\theta_1 = m_1 \int\limits_0^\infty dr \int\limits_0^{2\pi} \frac{e^{-\frac{r^2}{\lambda^2}} r}{r^2 + \kappa_0^2} e^{i\rho r \cos\varphi} d\varphi, \tag{29}$$

$$\theta_1 = m_1 2\pi \int\limits_0^\infty \frac{e^{-\frac{r^2}{\lambda^2}} r}{r^2 + \kappa_0^2} J_0(\rho r) dr, \tag{30}$$

After a simple transformation one can write Θ_1 in the form [17]:

$$\theta_1 = 2\pi m_1 K_0(\kappa_0 \rho) - 2\pi m_1 \int\limits_0^u e^{-\tau\kappa_0^2 - \frac{\rho^2}{4\tau}} \frac{1}{2\tau} d\tau, \tag{31}$$

where: $u = \lambda^{-2} \ll 1$,

$$\theta_2(x_0, x_2, x_3) = 2\pi m_1 \int\limits_0^\infty \frac{e^{-\frac{r^2}{\lambda^2}} r}{r^2 + \kappa_0^2} F(r^2, \ldots) J_0(\rho r) dr, \tag{32}$$

where:

$$F(r^2) = \frac{1}{\kappa \Delta} \left[a e^{-\kappa x_3} + b e^{-\kappa(1-x_3)} \right] \tag{33}$$

and

$$\kappa = \sqrt{r^2 + \frac{1}{4V_e^2}}. \tag{34}$$

The above integral expressions for Θ_1 and Θ_2 one can write as the definition formulae of the Bessel transform for functions [17]:

$$F_1(r) = \frac{e^{-\frac{r}{\lambda^2}}}{r^2 + \kappa_0^2}$$

and

$$F_2(r) = \frac{e^{-\frac{r}{\lambda^2}}}{r^2 + \kappa_0^2} F(r^2, \ldots).$$

For the case: $\kappa^2_0 = 1/(4V^2_0) - \gamma^2_0 > 0$, $\kappa^2_1 = |\kappa^2_0|$,

$$\theta_1 = m \int_{-\infty}^{\infty} \mathrm{d}q \int_{-\infty}^{\infty} \frac{\mathrm{e}^{\frac{-(\omega^2+q^2)}{\lambda^2}+\mathrm{i}\omega\rho}}{\omega^2 + q^2 - |\kappa_0^2|}\mathrm{d}\omega, \quad \text{see (28)}, \tag{35}$$

or

$$\theta_1 = m \int_{-\infty}^{\infty} \mathrm{d}q \int_{-\infty}^{\infty} \mathrm{d}\omega \mathrm{e}^{\mathrm{i}\omega q - u\kappa_1^2} \frac{\mathrm{e}^{-u(\omega^2+q^2-\kappa_1^2)}}{(\omega^2 + q^2 - \kappa_1^2)}. \tag{36}$$

Thus, we can write:

$$\theta_1 = m_1 \mathrm{e} \int_{-\infty}^{\infty} \mathrm{d}q \int_{-\infty}^{\infty} \mathrm{d}\omega \mathrm{e}^{\mathrm{i}\omega q} \Big[\int_u^0 \mathrm{e}^{-\nu(\omega^2+q^2-\kappa_1^2)}\mathrm{d}\nu + \frac{1}{(\omega^2 + q^2 - \kappa_1^2)} \Big]. \tag{37}$$

Using denotation:

$$\theta_1 = \theta_{11} + \theta_{12}, \tag{38}$$

$$\begin{aligned} \theta_{11} &= m_1 \mathrm{e}^{-u\kappa_1^2} \int_u^0 \mathrm{e}^{\nu\kappa_1^2}\mathrm{d}\nu \int_{-\infty}^{\infty} \mathrm{e}^{-\nu q^2}\mathrm{d}q \int_{-\infty}^{\infty} \mathrm{e}^{-\nu\omega^2+\mathrm{i}\omega\rho}\mathrm{d}\omega, \\ \theta_{12} &= m_1 \pi \mathrm{e}^{-u\kappa_1^2} \int_u^0 \mathrm{e}^{\nu\kappa_1^2} \frac{1}{\nu} \mathrm{e}^{-\frac{\rho^2}{4\nu}}\mathrm{d}\nu \end{aligned} \tag{39}$$

and finally, for Θ_{11} we obtain:

$$\theta_{11} = -m_1 \pi \int_0^u \mathrm{e}^{-\kappa_1^2(u-\nu)} \frac{1}{\nu} \mathrm{e}^{-\frac{\rho^2}{4\nu}}\mathrm{d}\nu, \tag{40}$$

for Q_{12} we have:

$$\begin{aligned} \theta_{12} &= m_1 \int_{-\infty}^{\infty} \mathrm{d}q \int_{-\infty}^{\infty} \mathrm{d}\omega \mathrm{e}^{i\omega\rho} \frac{1}{(\omega^2 + q^2 - \kappa_1^2)} \\ &= m_1 \pi \int_{-\infty}^{\infty} \frac{\mathrm{e}^{\mathrm{i}\omega\rho}}{\sqrt{\omega^2 - \kappa_1^2}} \mathrm{H}(|\omega| - \kappa_1)\mathrm{d}\omega \end{aligned} \tag{41}$$

and basing on [17], we have:

$$\theta_{12} = m_1 \pi^2 \mathrm{e}^{-u\kappa_1^2} \mathrm{N}_0(\kappa_1 \rho), \tag{42}$$

where $N_o(.)$ — Neumann function, H(.) — Heavisidea function.
Finally:

$$\theta_1 = -m_1\pi e^{-u\kappa_1^2}\Big[\int_0^u e^{\kappa_1^2\nu}\frac{1}{\nu}e^{-\frac{\rho^2}{4\nu}}d\nu + \pi N_0(\kappa_1\rho)\Big]. \tag{43}$$

Θ_2 one can obtain from equation (23), putting Θ_1 from (43) and Θ_{21}:

$$\theta_{21} = 2\pi m_1\int_0^\infty F(r^2,\ldots)J_0(\rho r)dr \tag{44}$$

and finally:

$$\theta = \int_{\mathbb{R}^2}\theta_1(x_0-\xi_0, x_2-\xi_2)\theta_{21}(\xi_0,\xi_2)d\xi_0 d\xi_2. \tag{45}$$

Since functions $F_1(r)$ and $F_2(r)$ are rapidly decaying functions with respect to the r variable, make the calculation of the integral effectiv. The numerical analysis and discussion of obtained relation we will present in the next paper.

REFERENCES

[1] ,*K. A. BOUNTING, G. CORNFIELD,* Toward a General Theory of Cutting: A Relationship Between the Incident Power Density and the Cut Speed, *Journal of Heet Transfer, 2, (1975), pp. 116-121..,*

[2] E. H. BREINAN, B. H. KEAR, C. H. BANAS, Physics today, 29, 11, (1976), p. 44.

[3] P. CANOVA, E. RAMOUS, J. Mater. Sci., 21, (1986), p. 2143

[4] C. W. DRAPER, J. M. POATE, Int. Metall. Rev., 30, (1985), p. 85.

[5] C. ESPOSITO, G. DAURELIO, A. CINGOLONI, Optics Lasers Engng., 3, (1982), pp. 139.

[6] J. N. GONSALVES, W. W. DULEY. *Cutting thin metal sheets with the cw CO^ laser,* J. Appl. Phys., Vol. 43, No 11, (1972), pp. 4684-4687.

[7] H. J. HEGGE, H. DE BEURS, J. NOSRDHUIS, J. TH. M. DE HOSSON, Metall. Trans., 21 A, (1990), pp. 987.

[8] *A. T. DE HOOP, A* modification of Cagniard's methods of molving seismic pulse problems, *Appl. Sci. Res., Sec. B, Vol. 18, (1961), pp. 349-356.*

[9] J. C. ION, H. R. SHERCLIFF, M. F. ASHBY, *Diagrams for Laser Materials Processing,* Acta Metall. Mater., Vol. 40, No 7, (1992), pp. 1539-1551.

[10] *A. LYNOW, B. H. BERKOWSKI,* Convection and thermal waves *(in Russian), Moscow 1974.*

[11] J. MAZUMDER, A. KAR, J. Metals, 39, 2, (1987), p. 18.

[12] J. MAZUMDER, J. Metals, 34, 7, (1982), p. 16.

[13] P. A. MOLIAN, Surface engineering, 2, (1986), p. 19.

[14] Y. NAMBA, Proc. Int. Conf. Laser Advanced Materials Processing, Osaka, Japan 1987, pp. 187-191.

[15] J. POWELL, Proc. Conf. Laser 5, IITT International, Gournay-sur-Marue, 1989, pp. 54-71.

[16] D. M. ROESSLER, in *Industrial Laser Annual Handbook* (edited by D. Belforte and M. Leoitt), Pern Well Bodis Tuba, 1986, pp. 16-30

[17] J. WATSON, *A treatise on the Theory of Bess el Functions,* (in Russian), London 1945.

[18] J. B. WILLIS, Proc. 1st Int. Conf. Lasers in Manufacturing, IFS, Bedford 1983, pp. 53-62

J. RAFA, S. KACZMAREK, W. PRZETAKIEWICZ, A. PAWLATA

Analiza rozkładu temperatury w warstwie metalu przy spawaniu laserowym

Streszczenie. Przedstawiono model matematyczny spawania laserowego warstwy metalowej. Sformułowano problem graniczny dla przyjętego modelu i skonstruowano (metoda transformacji całkowych) jego rozwiązania w postaci bezwymiarowej dla pełnego zakresu parametrów fizycznych i technicznych. Wyznaczono liczbę Verona determinującą jakościowy rozkład temperatury w warstwie. Otrzymane równanie służy do obliczania izoterm w materiale i umożliwia optyrnalizowanie procesów technologicznych laserowej obróbki metali.

И. РАФА, С. КАЧМАРЕК, В. ПжЕТАКИЕВИЧ, А. ПАВЛАТЛ

Анализ распределения температуры для обработки металлов лазерной сваркой

Резюме. Представлена математическа модель лазерной сварки металлического слоя. Формилированная предельная проблема для принятой модели и построен (методом интегральных преобразований) её решения в безразмерной форме дла полного диапазона физических и технических параметров.

Определено число Верона, в свою очередь определяющую качесвенное распределение температуры в слое.

Полученное уравнение полезное для вычисления изотерм в металле и делает возможным оптимизирование технологических процессов лазерной обработки металлов.